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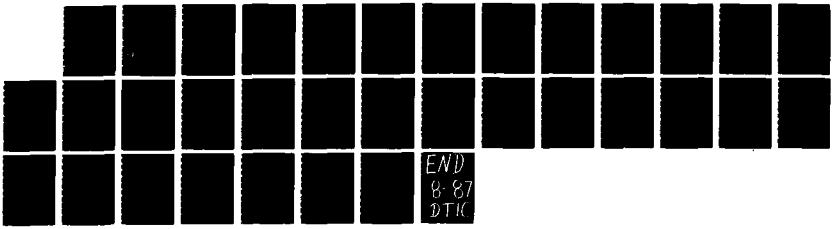

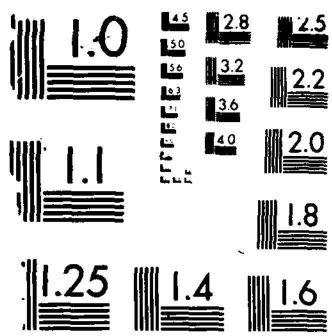

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TECHNICAL REPORT ARCCB-TR-87015

SOME RESULTS ON ORTHOTROPIC HIGH PRESSURE CYLINDERS

AD-A182 535

G. PETER O'HARA

JUNE 1987

US ARMY ARMAMENT RESEARCH, DEVELOPMENT
AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
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With the current emphasis on composite materials, it has become necessary to use the thick-wall cylinder equations for cylindrically orthotropic materials. This report is a preliminary investigation of these equations. The most important result is that many of the old ideas of high pressure cylinders will have to be changed. When the cylinder is very thin (wall ratio near 1.0), the simple thin-wall equations are adequate. However, as the thickness increases, stress variation through the wall becomes large and the full thick-wall

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20. ABSTRACT (CONT'D)

solution is necessary. In orthotropic (composite) cylinders this transition happens at a lower wall ratio. Furthermore, the maximum useful wall ratio may become smaller for a composite cylinder. (Keywords)

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

a = the inner radius

A = the material compliance matrix

b = the outer radius

C = the cylinder geometry parameter

D = simplified orthotropic material parameter

E_h = engineering modulus in the hoop direction

E_r = engineering modulus in the radial direction

h = a material constant for .al load equations

i = matrix row index

j = matrix column index

k = orthotropic material parameter

p = internal pressure

P = axial force

PP = plane-strain axial force

q = external pressure

r = radius

T = calculated constant for the axial load equations

W = wall ratio

x = radial direction

y = circumferential or hoop direction

z = axial direction

β_{ij} = compliance for the cylindrical problem

ϵ_i = strain in the i direction

σ_i = stress in the i direction

1 = matrix position for the x direction

2 = matrix position for the y direction

3 = matrix position for the z direction

[ ] = matrix of values

INTRODUCTION

The effective use of composites for high pressure cylinders requires that the thick-wall cylinder equations be rewritten to include cylindrically orthotropic material properties. This process adds several complicating factors to the analysis. First, is the addition of two more moduli of elasticity and two more Poisson's ratios to the material properties. Second, is that axial stress in the cylinder is no longer a constant in elastic analysis. In the new case an axial force will produce a correction to all three principal stresses which are a function of radius.

Several factors which simplify the analysis of cylinders are common to both the isotropic and orthotropic analysis. The principal stress directions correspond to the principal geometry directions of both the cylinder and the applied pressures. This fact eliminates shear effects from the analysis. The next simplification is that the geometry can be nondimensionalized by dividing by the internal radius and it is specified by the wall ratio ($W = b/a$). The two loads are internal and external pressure which are also nondimensional. In fact, stress can be specified as a function of either applied pressure when the ratio of internal to external pressure is a constant.

The form of the equations used was proposed by Voyiadjis, Kiousis, and Hartley (ref 1) in a 1985 paper dealing with the residual stresses in metal cylinders that have been subjected to large plastic deformation. However, it has been necessary to go back to the original work of Lekhnitskii (ref 2) and

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[1] G. Z. Voyiadjis, P. D. Kiousis, and C. S. Hartley, "Analysis of Residual Stresses in Cylindrically Anisotropic Materials," Experimental Mechanics, Vol. 25, No. 2, June 1985, pp. 145-147.

[2] S. G. Lekhnitskii, Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day Inc., San Francisco, 1963.

develop the equations more fully. In the metallic problem the difference in the engineering properties is not very great and some simplifying assumptions are possible. In contrast, the composite materials show a very large difference in properties with direction and the full set of equations is necessary. The assumptions that will be used in this report are that of a linear elastic cylindrically orthotropic material.

GEOMETRY

The problem is that of a long cylinder (Figure 1) with flat ends. The cylinder has a constant inner radius 'a' and outer radius 'b'. The ends are fully constrained in the axial direction or the classic plane-strain condition. The material is assumed to be cylindrically orthotropic with its principal axes coincident with the cylinder. This assumption provides for constant engineering properties in each of the three coordinate directions: radial, circumferential, and axial. While this report will only use the internal pressure load, the equations are written for constant pressure loads on either the internal or external surfaces. Also, this report will only use the plane-strain end condition, but will quote the equations for axial loads necessary to calculate other end load conditions.

Thick-Wall Equations

For this work, Hooke's Law is used in the following form:

$$[\epsilon] = [A][\sigma]$$

This must be modified for use in the cylindrical problem and a new material compliance matrix is generated in using the transformation equation

$$\beta_{ij} = A_{ij} - \frac{A_{i3}A_{3j}}{A_{33}}$$

In general, this new matrix must be transformed to supply the proper values as the function of the angular position in the cylinder. However, in the case of a cylindrically orthotropic material, this matrix has constant values.

In order to solve thick-wall cylinder problems for a variety of end load conditions, i.e., the generalized plane-strain, it is necessary to have two sets of equations. The first set of four equations is the plane-strain solution for a cylinder under a combination of internal and external pressure. The second set is for the cylinder with an axial load applied. With these equations the stresses can be calculated for any end load condition by superposition. Both sets of equations contain a single equation for stress in the radial, tangential, and axial directions. However, each set contains a fourth equation. In the pressure load set this equation calculates the total axial load generated by the plane-strain condition. In contrast, in the axial load set the fourth equation calculates a rather complicated constant used in the other three equations. The eight equations were taken from Reference 2 and corrected for several minor typographical errors, and all terms were eliminated which are zero for orthotropic materials. The resulting equations are given below at the 'a' equation in a pair (1a, 2a,...8a). The 'b' equation is the same equation in the form suggested by Reference 1. The conversion is done in two simple steps.

$$1 = \frac{b^{2k}}{b^{k-1}b^{k+1}} = \frac{b^{2k}b^{2k}}{b^{k-1}b^{k+1}b^{k-1}b^{k+1}}$$

Next, make the following substitution:

$$C_o = \frac{a}{b}$$

---

[1]G. Z. Voyiadjis, P. D. Kiousis, and C. S. Hartley, "Analysis of Residual Stresses in Cylindrically Anisotropic Materials," Experimental Mechanics, Vol 25, No. 2, June 1985, pp. 145-147.

[2]S. G. Lekhnitskii, Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day, Inc., San Francisco, 1963.

The equations for pressure load are:

$$\sigma_r = \frac{pa^{k+1} - qb^{k+1}}{b^{2k} - a^{2k}} r^{k-1} + \frac{qa^{k-1} - pb^{k-1}}{b^{2k} - a^{2k}} a^{k+1} b^{k+1} r^{-k-1} \tag{1a}$$

$$\sigma_r = \left[\frac{pC_o^{k+1} - q}{(1-C_o^{2k})} \right] \left(\frac{r}{b}\right)^{k-1} + \left[\frac{qC_o^{k-1} - p}{(1-C_o^{2k})} \right] C_o^{k+1} \left(\frac{b}{r}\right)^{k+1} \tag{1b}$$

$$\sigma_\theta = \frac{pa^{k+1} - qb^{k+1}}{b^{2k} - a^{2k}} kr^{k-1} - \frac{qa^{k-1} - pb^{k-1}}{b^{2k} - a^{2k}} ka^{k+1} b^{k+1} r^{-k-1} \tag{2a}$$

$$\sigma_\theta = \left[\frac{pC_o^{k+1} - q}{(1-C_o^{2k})} \right] k\left(\frac{r}{b}\right)^{k-1} - \left[\frac{qC_o^{k-1} - p}{(1-C_o^{2k})} \right] kC_o^{k+1} \left(\frac{b}{r}\right)^{k+1} \tag{2b}$$

$$\sigma_z = -\frac{1}{A_{33}} \left[\frac{pa^{k+1} - qb^{k+1}}{b^{2k} - a^{2k}} (A_{13}+kA_{23}) r^{k-1} + \frac{qa^{k-1} - pb^{k-1}}{b^{2k} - a^{2k}} a^{k+1} b^{k+1} (A_{13}-kA_{23}) r^{-k-1} \right] \tag{3a}$$

$$\sigma_z = \frac{-1}{A_{33}} \left[\frac{(pC_o^{k+1}-q)}{(1-C_o^{2k})} \right] (A_{13}+kA_{23}) \left(\frac{r}{b}\right)^{k-1} + \left[\frac{qC_o^{k-1} - p}{(1-C_o^{2k})} \right] (A_{13}-kA_{23}) C_o^{k+1} \left(\frac{b}{r}\right)^{k+1} \tag{3b}$$

$$PP = -\frac{2\pi}{A_{33}(b^{2k}-a^{2k})} \left[(qb^{k+1}-pa^{k+1})(b^{k+1}-a^{k+1}) \frac{A_{13} + kA_{23}}{1 + k} + (qa^{k-1}-pb^{k-1})(b^{k-1}-a^{k-1}) a^2 b^2 \cdot \frac{A_{13} - kA_{23}}{1 - k} \right] \tag{4a}$$

$$PP = \frac{2\pi}{A_{33}(1-C_o^{2k})} \left[b^2(q-PC_o^{k+1})(1-C_o^{k+1}) \frac{A_{13} + kA_{23}}{1 + k} \right.$$

$$\left. + a^2(qC_o^{k-1}-P)(1-C_o^{k-1}) \frac{A_{13} - kA_{23}}{1 - k} \right] \tag{4b}$$

The equations for axial load are:

$$\sigma_r = \frac{Ph}{T} (1 - \frac{b^{k+1} - a^{k+1}}{b^{2k} - a^{2k}} r^{k-1} - \frac{b^{k-1} - a^{k-1}}{b^{2k} - a^{2k}} a^{k+1}b^{k+1}r^{-k-1}) \tag{5a}$$

$$\sigma_r = \frac{Ph}{T} \left[1 - \frac{(1-C_o^{k+1})}{(1-C_o^{2k})} (\frac{r}{b})^{k-1} - \frac{(1-C_o^{k-1})}{(1-C_o^{2k})} C_o^{k+1}(\frac{b}{r})^{k+1} \right] \tag{5b}$$

$$\sigma_\theta = \frac{Ph}{T} (1 - \frac{b^{k+1} - a^{k+1}}{b^{2k} - a^{2k}} kr^{k-1} + \frac{b^{k-1} - a^{k-1}}{b^{2k} - a^{2k}} ka^{k+1}b^{k+1}r^{-k-1}) \tag{6a}$$

$$\sigma_\theta = \frac{Ph}{T} (1 - \frac{(1-C_o^{k+1})}{(1-C_o^{2k})} k(\frac{r}{b})^{k-1} + \frac{(1-C_o^{k-1})}{(1-C_o^{2k})} kC_o^{k+1}(\frac{b}{r})^{k+1}) \tag{6b}$$

$$\sigma_z = \frac{P}{T} - \frac{Ph}{TA_{33}} \left[A_{13} + A_{23} - \frac{b^{k+1} - a^{k+1}}{b^{2k} - a^{2k}} (A_{13}+kA_{23})r^{k-1} \right.$$

$$\left. - \frac{b^{k-1} - a^{k-1}}{b^{2k} - a^{2k}} a^{k+1}b^{k+1}(A_{13}-kA_{23})r^{-k-1} \right] \tag{7a}$$

$$\sigma_z = \frac{P}{T} - \frac{Ph}{TA_{33}} \left[A_{13} + A_{23} - \frac{(1-C_o^{k-1})}{(1-C_o^{2k})} (A_{13}+kA_{23})(\frac{r}{b})^{k-1} \right.$$

$$\left. - \frac{(1-C_o^{k-1})}{1 - C_o^{2k}} (A_{13}-kA_{23})C_o^{k+1}(\frac{b}{r})^{k+1} \right] \tag{7b}$$

$$T = \pi(b^2-a^2) - \frac{2\pi h}{A_{33}} \left[\frac{b^2 - a^2}{2} (A_{13}+A_{23}) - \frac{(b^{k+1}-a^{k+1})^2}{b^{2k} - a^{2k}} \cdot \frac{A_{13} + kA_{23}}{k + 1} \right.$$

$$\left. - \frac{(b^{k-1}-a^{k-1})^2 a^2 b^2}{b^{2k} - a^{2k}} \cdot \frac{A_{13} - kA_{23}}{k - 1} \right] \tag{8a}$$

$$T = \pi(b^2-a^2) - \frac{2\pi h}{A_{33}} \left[\frac{b^2 - a^2}{2} (A_{13}+A_{23}) - \frac{b^2(1-C_o{}^{k+1})^2}{(1-C_o{}^{2k})} \frac{(A_{13}-kA_{23})}{(k+1)} \right.$$

$$\left. - \frac{a^2(1-C_o{}^{k-1})^2}{(1-C_o{}^{2k})} \frac{(A_{13}-kA_{23})}{(k-1)} \right] \tag{8b}$$

The two common material parameters are:

$$k = \sqrt{\frac{\beta_{11}}{\beta_{22}}} , \quad h = \frac{A_{23} - A_{13}}{\beta_{11} - \beta_{22}}$$

These equations have been implemented as four FORTRAN subprograms: a function to set up the material constants and return simplified material parameter for printout, a subroutine to calculate the plane-strain force, a subroutine to return pressure stresses, and a subroutine to return stresses due to axial loads (see Appendix A). These can be incorporated easily into any other stress analysis program.

RESULTS

The new solution has been used to produce three plots. In each of these plots a parameter is plotted versus a wall ratio range of 1.0 to 2.5 and is calculated for the plane-strain end condition. The three parameters are solution error, growth potential, and failure pressure for failure in simple tension. The hoop stress (circumferential) at the bore is used in all cases because this is the critical stress in thick cylinders. Solution error is the

percent error when compared with the standard strength of materials solution for thin cylinders. Growth potential is the percent reduction in hoop stress when the wall is increased in thickness by one percent. The failure pressure assumes that the tube will fail in hoop tension at a stress of 100,000 pressure units. There are five curves on each plot for five values of the orthotropic material parameter D. D is the ratio defined by dividing the engineering modulus in the hoop direction by the modulus in the radial direction. For D equal to one, the tube material is isotropic and for D equal to 128, the material is highly orthotropic.

DISCUSSION

All three of these plots show the potential for serious problems with thick-wall composite tubes. The problems result from the low radial stiffness of the tubes relative to the higher hoop stiffness. It appears that the ratio of hoop stiffness divided by radial stiffness ($D = E_h/E_r$) is an important design parameter. As this ratio becomes larger, the tube has diminished ability to transfer load in the radial direction to the outer fibers. Therefore, the outer fibers may not have the ability to carry load as well as the material in the same position in an isotropic material.

CONCLUSION

The behavior of an orthotropic high pressure cylinder may be very different from that of an isotropic cylinder. Therefore, any composite design must proceed with extreme care. However, these equations must first be verified (Appendix B).

REFERENCES

1. G. Z. Voyiadjis, P. D. Kiousis, and C. S. Hartley, "Analysis of Residual Stresses in Cylindrically Anisotropic Materials," Experimental Mechanics, Vol. 25, No. 2, June 1985, pp. 145-147.

2. S. G. Lekhnitskii, Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day Inc., San Francisco, 1963.

B-1. V. Verderaime, "Development of In Situ Stiffness Properties For Shuttle Booster Filament Wound Case," NASA Technical Paper 2377, George C. Marshall Space Flight Center, Huntsville, AL, August 1984.

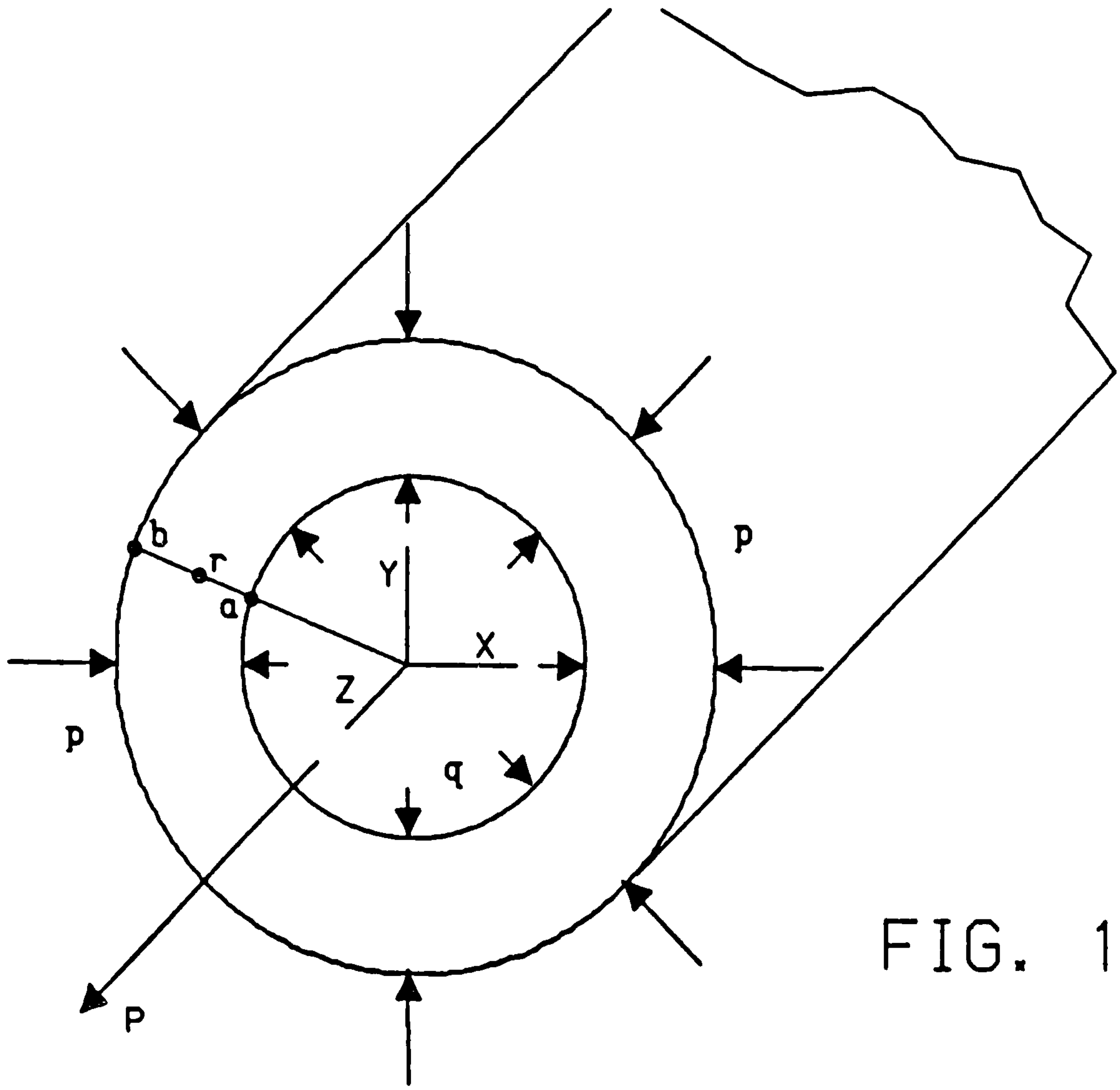

FIG. 1

Cylinder Geometry

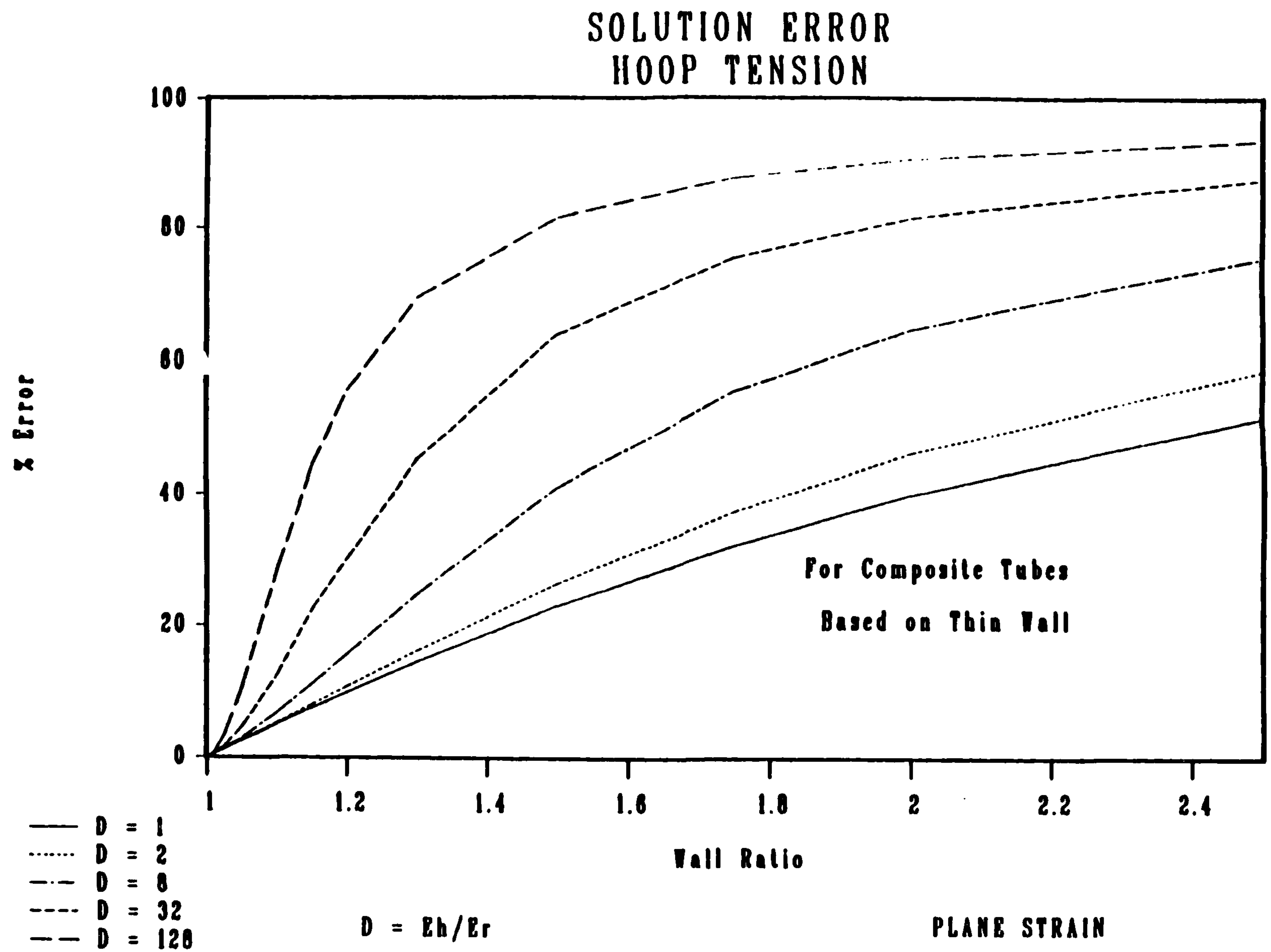
SOLUTION ERROR
HOOP TENSION
% Error
100
80
60
40
20
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
Wall Ratio
For Composite Tubes
Based on Thin Wall
D = 1
D = 2
D = 8
D = 32
D = 128
D = Eh/Er
PLANE STRAIN

Figure 2

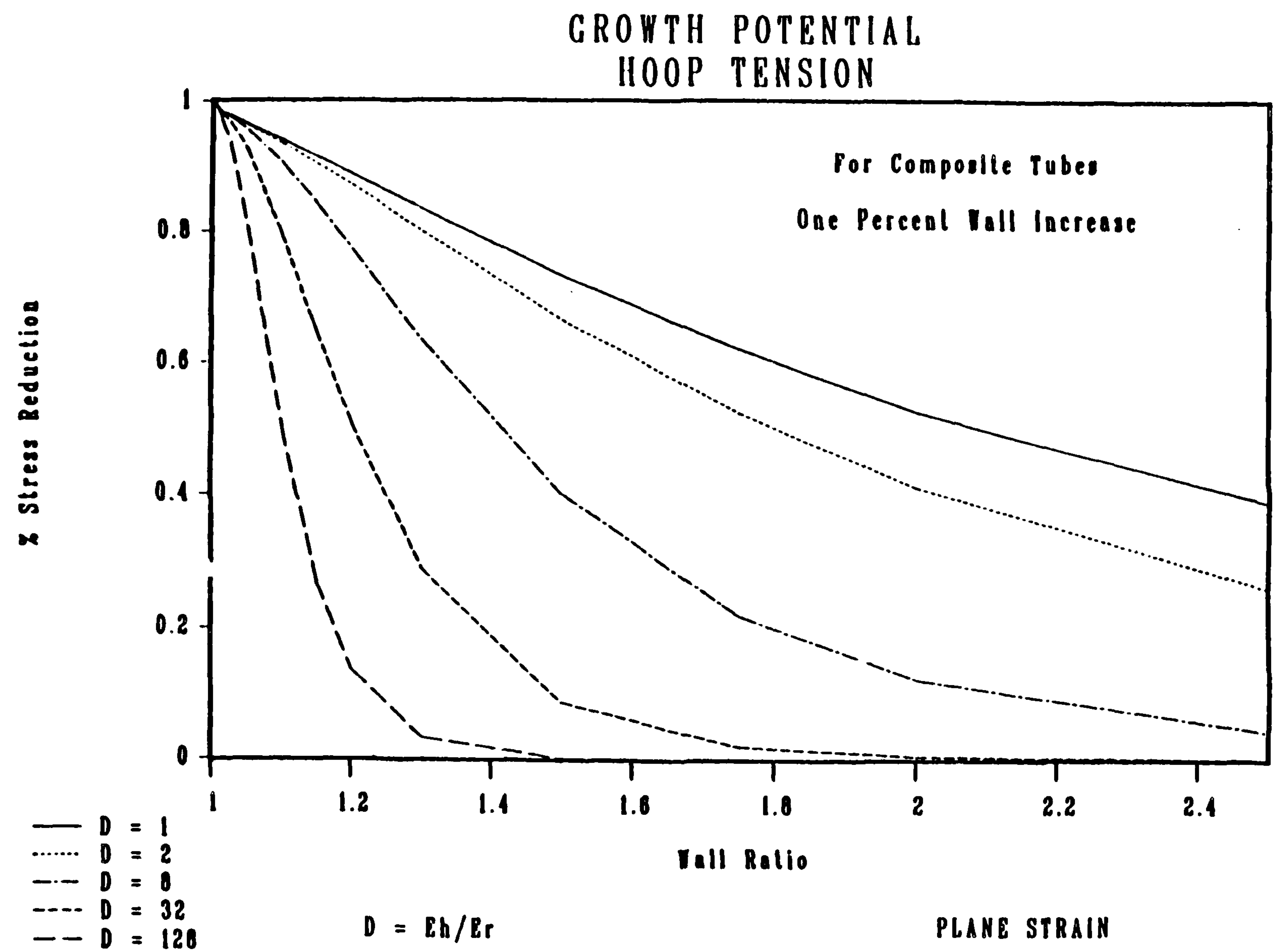
GROWTH POTENTIAL
HOOP TENSION
For Composite Tubes
One Percent Wall Increase
% Stress Reduction
Wall Ratio
1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
D = 1
D = 2
D = 8
D = 32
D = 128
D = Eh/Er
PLANE STRAIN

Figure 3

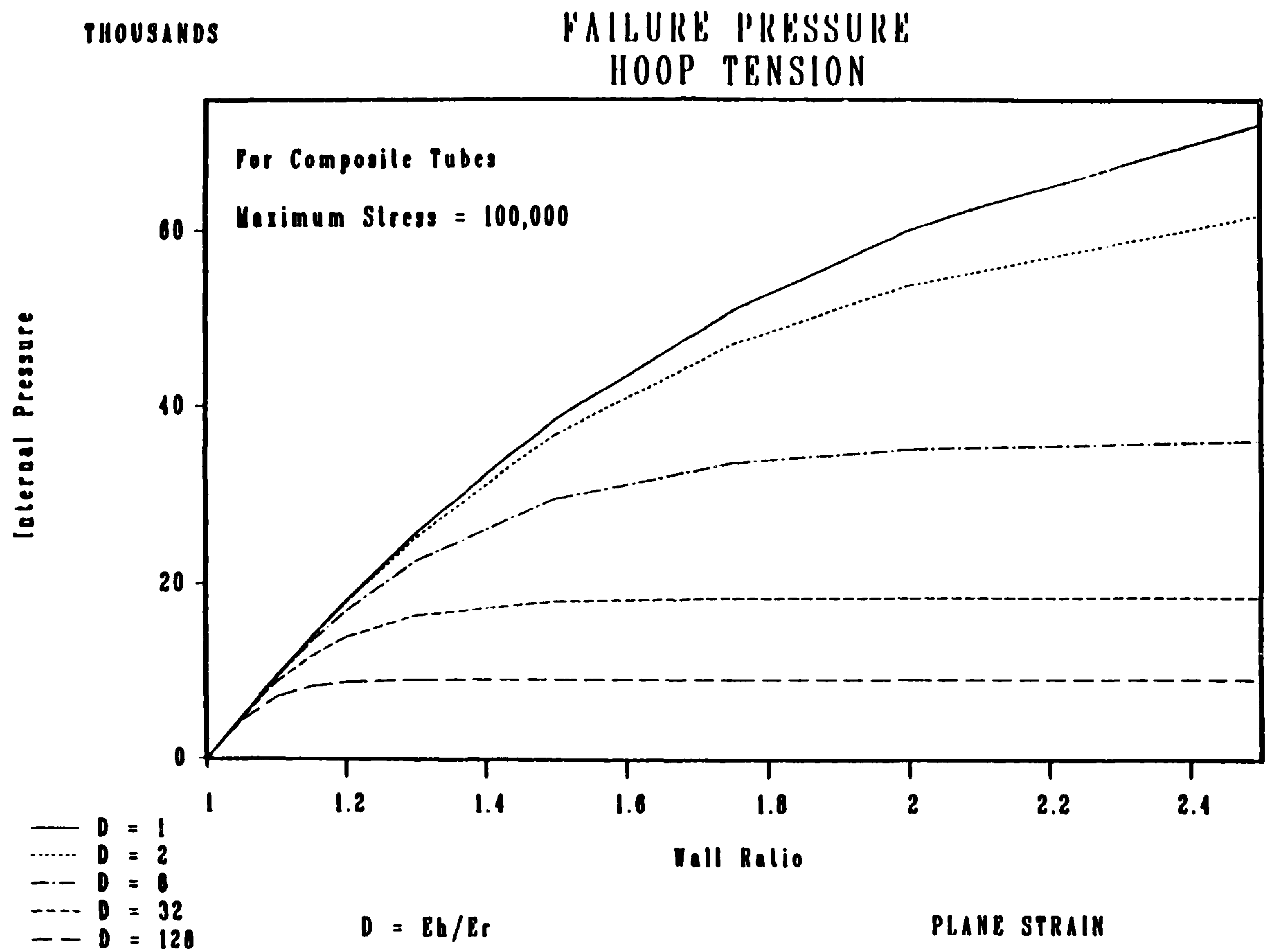
THOUSANDS
FAILURE PRESSURE
HOOP TENSION
For Composite Tubes
Maximum Stress = 100,000
60
40
20
0
Internal Pressure
1
1.2
1.4
1.6
1.8
2
2.2
2.4
Wall Ratio
D = 1
D = 2
D = 8
D = 32
D = 128
D = Eh/Er
PLANE STRAIN

Figure 4

APPENDIX A

SOME RESULTS ON ORTHOTROPIC HIGH PRESSURE CYLINDERS

The following four subprograms calculate stresses in a cylindrically orthotropic cylinder:

ORCYMA - is a function that sets up the material constants and returns a single parameter for use in output identification. (This must be called with each change of material.)

ORCYFO - is a subroutine that calculates the axial force due to the plane-strain end condition.

ORCYPR - is a subroutine that calculates stresses due to:

(a) internal pressure

(b) external pressure

(c) twisting about the axis

ORCYTE - is a subroutine that calculates stresses due to an axial force.

```
C
C     - ORCYMA -
C
C    INPUT
C    IMAT = POINTER TO THE CURRENT MATERIAL COMPLIANCE MATRIX
C    STRESS = PLANE STRESS ANALYSIS ? (LOGICAL)
C
C    IN THE /MATERL/ COMMON BLOCK
C    AA = A SET OF MATERIAL COMPLIANCE PROPERTIES
C
C    OUTPUT
C    ORCYMA = A SIMPLIFIED MATERIAL CONSTANT FOR PRINTER OUTPUT ONLY
C
C    IN THE /MATERL/ COMMON BLOCK
C    THE MATERIAL CONSTANTS
C    K - K2 - KP1 - KN1 - APKA - AMKA - A33 - A13P23 - H
C
      FUNCTION ORCYMA(IMAT,STRESS)
      IMPLICIT REAL*8(A-H,K-Z)
      LOGICAL STRESS
      COMMON /MATERL/K,K2,KP1,KM1,APKA,AMKA,A33,A13P23,H,AA(6,6,15)
C
C   SET UP THE MATERIAL CONSTANTS
C
      A11=AA(1,1,IMAT)
      A22=AA(2,2,IMAT)
      A13=AA(1,3,IMAT)
      A23=AA(2,3,IMAT)
      A33=AA(3,3,IMAT)
      IF(.NOT.STRESS)THEN
      B11=A11-A13*A13/A33
      B22=A22-A23*A23/A33
      ELSE
      B11=A11
      B22=A22
      ENDIF
      D=B11/B22
      K=DSQRT(D)
      KM1=K-1.0 D0
      KP1=K+1.0 D0
      K2=K*2.0D0
      APKA=A13+K*A23
      AMKA=A13-K*A23
      A13P23=A13+A23
      H=0.0 D0
      IF(B11.NE.B22)H=(A23-A13)/(B11-B22)
      ORCYMA=D
      RETURN
      END
```

```
C
C     - ORCYFO -
C
C    INPUT
C    A = INNER RADIUS
C    B = OUTER RADIUS
C    PIN = INNER PRESSURE
C    POUT = OUTER PRESSURE
C
C    OUTPUT
C    PP = PLANE STRAIN FORCE
C
      SUBROUTINE ORCYFO(A,B,PIN,POUT,PP)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON /MATERL/K,K2,KP1,KM1,APKA,AMKA,A33,A13P23,H
      DATA PI/3.14159259 D0/
C
C   CALCULATE THE TOTAL AXIAL FORCE FOR PLANE STRAIN
C
      CO=A/B
      PP=B*B*(POUT-PIN*CO**KP1)*(1.0D0-CO**KP1)*APKA/(1.0D0+K)
      IF(K.NE.1.0D0)THEN
      PP=PP+A*A*(POUT*CO**KM1-PIN)*(1.0D0-CO**KM1)*AMKA/(1.0D0-K)
      ENDIF
      PP=(2.0D0*PI/(A33*(1.0D0-CO**K2)))*PP
      RETURN
      END
```

```
C
C      - ORCYPR -
C
C     INPUT
C     A = INNER RADIUS
C     B = OUTER RADIUS
C     R = LOCAL RADIUS
C     PIN = INNER PRESSURE
C     POUT = OUTER PRESSURE
C     M = TWISTING MOMENT
C
C     OUTPUT
C     SIG = PRINCIPAL STRESSES
C
      SUBROUTINE ORCYPR(A,B,R,PIN,POUT,M,SIG)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON /MATERL/K,K2,KP1,KM1,APKA,AMKA,A33,A13P23,H
      DIMENSION SIG(4)
      DATA PI/3.14159259 D0/
C
C   CALCULATE THE STRESSES FROM INTERNAL PRESSURE, EXTERNAL PRESSURE
C     AND TORSION ABOUT THE AXIS
C
      C0=A/B
C RADIAL STRESS
      SIG(1)    =((PIN*C0**KP1-POUT)/(1.0D0-C0**K2))*((R/B)**KM1)
     A          +((POUT*C0**KM1-PIN)/(1.0D0-C0**K2))*C0**KP1*(B/R)**KP1
C TANGENTIAL STRESS
      SIG(2)    =((PIN*C0**KP1-POUT)/(1.0D0-C0**K2))*K*((R/B)**KM1)
     A          -((POUT*C0**KM1-PIN)/(1.0D0-C0**K2))*K*C0**KP1*(B/R)**KP1
C AXIAL STRESS
      SIG(3)=((PIN*C0**KP1-POUT)/(1.0D0-C0**K2))*APKA*(R/B)**KM1
      SIG(3)=SIG(3)+((POUT*C0**KM1-PIN)/(1.0D0-C0**K2))*AMKA*C0**KP1
     A *(B/R)**KP1
      SIG(3)=-SIG(3)/A33
C SHEAR STRESS
      SIG(4)=2.0D0*M*R/(PI*B**4*(1.0D0-C0**4))
      RETURN
      END
```

```
C
C     - ORCYTE -
C
C    INPUT
C    A = INNER RADIUS
C    B = OUTER RADIUS
C    R = LOCAL RADIUS
C    P = TOTAL APPLIED FORCE
C
C    OUTPUT
C    SIG = PRINCIPAL STRESSES
C
      SUBROUTINE ORCYTE(A,B,R,P,SIG)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON /MATERL/K,K2,KP1,KM1,APKA,AMKA,A33,A13P23,H
      DIMENSION SIG(3)
      DATA PI/3.14159259 D0/
      DATA AOLD,BOLD,KOLD,A3LD/4*0.0D0/
C
C   CALCULATE THE STRESSES FROM AXIAL TENSION
C
      IF(K.EQ.1.0D0)THEN
      SIG(1)=0.0D0
      SIG(2)=0.0D0
      SIG(3)=P*1.0D0/(PI*(B*B-A*A))
      GO TO 10
      ENDIF
      C0=A/B
      IF(A.NE.AOLD.OR.B.NE.BOLD.OR.K.NE.KOLD.OR.A33.NE.A3LD)THEN
      T=(B*B-A*A)*A13P23/2.0D0
      T=T-B*B*(((1.0D0-C0**KP1)**2/(1.0D0-C0**K2))*(APKA/(K+1.0D0)))
      T=T-A*A*(((1.0D0-C0**KM1)**2/(1.0D0-C0**K2))*(AMKA/(K-1.0D0)))
      T=PI*(B*B-A*A)-(2.0D0*PI*H/A33)*T
      AOLD=A
      BOLD=B
      KOLD=K
      A3LD=A33
      ENDIF
C
      SIG(1)=1.0D0-((1.0D0-C0**KP1)/(1.0D0-C0**K2))*(R/B)**KM1
      SIG(1)=SIG(1)-((1.0D0-C0**KM1)/(1.0D0-C0**K2))*C0**KP1*(B/R)**KP1
      SIG(1)=(H/T)*SIG(1)*P
C
      SIG(2)=1.0D0-((1.0D0-C0**KP1)/(1.0D0-C0**K2))*K*(R/B)**KM1
      SIG(2)=SIG(2)+((1.0D0-C0**KM1)/(1.0D0-C0**K2))*K*C0**KP1*
     A     (B/R)**KP1
      SIG(2)=(H/T)*SIG(2)*P
C
      SIG(3)=A13P23-((1.0D0-C0**KP1)/(1.0D0-C0**K2))*APKA*(R/B)**KM1
      SIG(3)=SIG(3)-((1.0D0-C0**KM1)/(1.0D0-C0**K2))*AMKA*C0**KP1*
     A  (B/R)**KP1
      SIG(3)=(1.0D0/T-(H/(T*A33))*SIG(3))*P
   10 RETURN
      END
```

APPENDIX B

SOME RESULTS ON ORTHOTROPIC HIGH PRESSURE CYLINDERS

SOLUTION VERIFICATION

This solution was verified by comparison with a ten-element finite element solution using the ABAQUS (4.5-175) code. ABAQUS is a general nonlinear program supplied by:

Hibbitt, Karlsson, and Sorensen, Inc.
101 Medway Street
Providence, RI 02906
(401) 861-0820

For this problem only the linear portion of ABAQUS was used in combination with an eight node axisymmetric element (CAX8) and a linear elastic orthotropic material definition. It was necessary to use only a single row of ten elements for all solutions and the stresses were recovered at the nodal points. The proper use of nodal point constraints and constraint equations insured that the radial faces of the model remained flat and parallel.

The material properties were that of a graphite epoxy wrapped composite cylinder. The material was used in a major program at NASA (ref B-1). For convenience, the theoretical analysis program was written to use the same material definition record as ABAQUS. This also insured that both programs used exactly the same material information.

A total of 18 ABAQUS solutions were generated for a selection of three different wall ratios, four different loading conditions and two fiber wrap angles (material properties). Four solutions have been selected and the results are shown in Tables B-1 through B-4. The four cases were selected for no particular reason except that each of the four loads is represented.

[B-1] V. Verderaime, "Development of In Situ Stiffness Properties For Shuttle Booster Filament Wound Case," NASA Technical Paper 2377, George C. Marshall Space Flight Center, Huntsville, AL, August 1984.

TABLE 8-1

ORTHOTROPIC HIGH PRESSURE CYLINDER THEORY

SOLUTION COMPARISON WITH ABAQUS USING 10 CAX8 ELEMENTS

FOR A GRAPHITE EPOXY OPEN END CYLINDER

WALL RATIO = 1.50 - WRAP ANGLE = 0.0 DEGREES

APPLIED LOADS
INTERNAL PRESSURE = 1.0
EXTERNAL PRESSURE = 0.0
AVERAGE AXIAL STRESS = 0.0

MATERIAL STIFFNESS MATRIX

	XX	ZZ	YY		XZ	XY	ZY
XX	8.298E+9	3.586E+9	3.791E+9	-	0.000E+0	0.000E+0	0.000E+0
ZZ	3.586E+9	8.298E+9	3.791E+9	-	0.000E+0	0.000E+0	0.000E+0
YY	3.791E+9	3.791E+9	122.4E+9	-	0.000E+0	0.000E+0	0.000E+0
- -	- - - -	- - - -	- - - -	-	- - - -	- - - -	- - -
XZ	0.000E+0	0.000E+0	0.000E+0	-	2.550E+9	0.000E+0	0.000E+0
XY	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	6.540E+9	0.000E+0
ZX	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	0.000E+0	6.540E+9

CALCULATED STRESSES

	RADIAL STRESS		HOOP STRESS		AXIAL STRESS	
RADIUS	THEORY	ABAQUS	THEORY	ABAQUS	THEORY	ABAQUS
1.000	-1.000	-.994	4.330	4.33	-.253	-.250
1.050	-.768	-.762	3.474	3.48	-.170	-.167
1.100	-.590	-.586	2.833	2.83	-.106	-.104
1.150	-.452	-.449	2.347	2.35	-.056	-.055
1.200	-.344	-.342	1.978	1.98	-.016	-.016
1.250	-.257	-.255	1.695	1.70	0.015	0.016
1.300	-.186	-.185	1.478	1.48	0.041	0.042
1.350	-.128	-.127	1.313	1.31	0.063	0.063
1.400	-.078	-.078	1.188	1.19	0.082	0.082
1.450	-.036	-.036	1.094	1.09	0.098	0.098
1.500	0.000	0.000	1.026	1.03	0.112	0.112

TABLE 3-2

ORTHOTROPIC HIGH PRESSURE CYLINDER THEORY

SOLUTION COMPARISON WITH ABAQUS USING 10 CAX8 ELEMENTS

FOR A GRAPHITE EPOXY CYLINDER WITH END LOADS

WALL RATIO = 1.75 - WRAP ANGLES = 20.0,-20.0 DEGREES

APPLIED LOADS
INTERNAL PRESSURE = 0.0
EXTERNAL PRESSURE = 0.0
AVERAGE AXIAL STRESS = 1.0

MATERIAL STIFFNESS MATRIX

	XX	ZZ	YY	-	XZ	XY	ZY
XX	8.298E+9	3.610E+9	3.767E+9	-	0.000E+0	0.000E+0	0.000E+0
ZZ	3.610E+9	11.63E+9	13.81E+9	-	0.000E+0	0.000E+0	0.000E+0
YY	3.767E+9	13.81E+9	99.05E+9	-	0.000E+0	0.000E+0	0.000E+0
-	-	-	-	-	-	-	-
XZ	0.000E+0	0.000E+0	0.000E+0	-	3.017E+9	0.000E+0	0.000E+0
XY	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	6.073E+9	0.000E+0
ZY	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	0.000E+0	16.56E+9

CALCULATED STRESSES

	RADIAL STRESS		HOOP STRESS		AXIAL STRESS	
RADIUS	THEORY	ABAQUS	THEORY	ABAQUS	THEORY	ABAQUS
1.000	0.000	0.002	0.733	0.734	1.078	1.08
1.075	0.043	0.044	0.503	0.504	1.065	1.07
1.150	0.067	0.068	0.328	0.329	1.053	1.05
1.225	0.073	0.079	0.187	0.188	1.040	1.04
1.300	0.081	0.082	0.067	0.068	1.026	1.03
1.375	0.077	0.078	-.040	0.040	1.011	1.01
1.450	0.067	0.069	-.140	0.140	0.995	0.995
1.525	0.056	0.056	-.236	0.236	0.978	0.978
1.600	0.040	0.040	-.332	0.332	0.960	0.960
1.675	0.021	0.021	-.428	0.428	0.941	0.941
1.750	0.000	0.000	-.526	0.526	0.921	0.921

TABLE B-3

ORTHOTROPIC HIGH PRESSURE CYLINDER THEORY

SOLUTION COMPARISON WITH ABAQUS USING 10 CAX8 ELEMENTS

FOR A GRAPHITE EPOXY PLANE STRAIN CYLINDER

WALL RATIO = 2.00 - WRAP ANGLE = 0.0 DEGREES

APPLIED LOADS
INTERNAL PRESSURE = 0.0
EXTERNAL PRESSURE = 1.0
AVERAGE AXIAL STRESS = 0.0

MATERIAL STIFFNESS MATRIX

	XX	ZZ	YY		XZ	XY	ZY
XX	8.298E+9	3.586E+9	3.791E+9	-	0.000E+0	0.000E+0	0.000E+0
ZZ	3.586E+9	8.298E+9	3.791E+9	-	0.000E+0	0.000E+0	0.000E+0
YY	3.791E+9	3.791E+9	122.4E+9	-	0.000E+0	0.000E+0	0.000E+0
XZ	0.000E+0	0.000E+0	0.000E+0	-	2.550E+9	0.000E+0	0.000E+0
XY	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	6.540E+9	0.000E+0
ZX	0.000E+0	0.000E+0	0.000E+0	-	0.000E+0	0.000E+0	6.540E+9

CALCULATED STRESSES

	RADIAL STRESS		HOOP STRESS		AXIAL STRESS	
RADIUS	THEORY	ABAQUS	THEORY	ABAQUS	THEORY	ABAQUS
1.000	0.000	-.002	-1.078	-1.08	-.019	-.020
1.100	-.095	-.097	-1.046	-1.05	-.059	-.060
1.200	-.177	-.178	-1.127	-1.13	-.095	-.096
1.300	-.256	-.256	-1.287	-1.29	-.132	-.132
1.400	-.337	-.337	-1.507	-1.51	-.170	-.170
1.500	-.424	-.424	-1.780	-1.78	-.211	-.211
1.600	-.519	-.518	-2.103	-2.10	-.257	-.257
1.700	-.623	-.622	-2.474	-2.47	-.308	-.309
1.800	-.737	-.736	-2.892	-2.89	-.364	-.364
1.900	-.862	-.861	-3.360	-3.36	-.426	-.425
2.000	1.000	0.999	-3.878	-3.88	-.493	-.493

TABLE B-4

ORTHOTROPIC HIGH PRESSURE CYLINDER THEORY

SOLUTION COMPARISON WITH ABAQUS USING 10 CAX8 ELEMENTS

FOR A GRAPHITE EPOXY PLANE STRAIN CYLINDER

WALL RATIO = 2.00 - WRAP ANGLE = 0.0 DEGREES

APPILIED LOADS
INTERNAL PRESSURE = 1.0
EXTERNAL PRESSURE = 1.0
AVERAGE AXIAL STRESS = 0.0

MATERIAL STIFFNESS MATRIX

	XX	ZZ	YY	XZ	XY	ZY
XX	8.298E+9	3.586E+9	3.791E+9	0.000E+0	0.000E+0	0.000E+0
ZZ	3.586E+9	8.298E+9	3.791E+9	0.000E+0	0.000E+0	0.000E+0
YY	3.791E+9	3.791E+9	122.4E+9	0.000E+0	0.000E+0	0.000E+0
XZ	0.000E+0	0.000E+0	0.000E+0	2.550E+9	0.000E+0	0.000E+0
XY	0.000E+0	0.000E+0	0.000E+0	0.000E+0	6.540E+9	0.000E+0
ZX	0.000E+0	0.000E+0	0.000E+0	0.000E+0	0.000E+0	6.540E+9

CALCULATED STRESSES

RADIUS	RADIAL STRESS THEORY	RADIAL STRESS ABAQUS	HOOP STRESS THEORY	HOOP STRESS ABAQUS	AXIAL STRESS THEORY	AXIAL STRESS ABAQUS
1.000	1.000	-.981	2.801	2.81	-.374	-.366
1.100	-.723	-.709	1.412	1.42	-.281	-.276
1.200	-.585	-.577	0.501	0.505	-.229	-.226
1.300	-.528	-.523	-.163	-.161	-.227	-.225
1.400	-.522	-.519	-.701	-.699	-.234	-.232
1.500	-.550	-.548	-1.179	-1.18	-.254	-.253
1.600	-.603	-.602	-1.635	-1.63	-.285	-.284
1.700	-.678	-.676	-2.093	-2.09	-.325	-.324
1.800	-.769	-.768	-2.563	-2.57	-.372	-.371
1.900	-.877	-.876	-3.071	-3.07	-.427	-.425
2.000	1.000	0.999	-3.609	-3.61	-.489	-.488

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